EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. PG is the normal to a standard ellipse at P, G being on the major axis. GP is produced outwards to Q so that PQ = GP. Show that the locus of Q is an ellipse whose eccentricity is $\frac{a^2-b^2}{a^2+b^2}$.

Sol.

2. P & Q are the corresponding points on a standard ellipse & its auxiliary circle. The tangent at P to the ellipse meets the major axis in T. Prove that QT touches the auxiliary circle.

Sol.

3. The point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is joined to the ends A,A′ of the major axis. If the lines through perpendicular to PA, PA′ meet the major axis in Q and R then prove that ℓ(QR) = length of latus rectum.

Sol.

4. Given the equation of the ellipse $\frac{(x-3)^2}{16}+\frac{(y+4)^2}{49}=1$, a parabola is such that its vertex is the lowest point of the ellipse and it passes through the ends of the minor axis of the ellipse. The equation of the parabola is in the form $16y = a(x-h)^2 - k$. Determine the value of $(a + h + k)$.

Sol.

5. A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches at the point P on it in the first quadrant & meets the coordinate axes in A & B respectively. If P divides AB in the ratio 3 : 1 reckoning from the x-axis find the equation of the tangent.

Sol.

6. Consider an ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ with centre C and a point P on it with eccentric angle $\frac{\pi}{4}$. Normal drawn at P intersects the major and minor axes in A and B respectively. N_1 and N_2 are the feet of the perpendiculars from the foci S_1 and S_2 respectively on the tangent at P and N is the foot of the perpendicular from the centre of the ellipse on the normal at P. Tangent at P intersects the axis of x at T. Match the entries of Column-I with the entries of Column-II.

Column-I	**Column-II**
(A) (CA)(CT) is equal to	(P) 9
(B) (PN)(PB) is equal to	(Q) 16
(C) $(S_1N_1)(S_2N_2)$ is equal to	(R) 17
(D) $(S_1P)(S_2P)$ is equal to	(S) 25

Sol.

7. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P & Q. Prove that the tangents at P & Q of the ellipse $x^2 + 2y^2 = 6$ are at right angles.

Sol.

8. Rectangle ABCD has area 200. An ellipse with area 200π passes through A and C and has foci at B and D. Find the perimeter of the rectangle.

Sol.

9. Consider the parabola $y^2 = 4x$ and the ellipse $2x^2 + y^2 = 6$, intersecting at P and Q.
(a) Prove that the two curves are orthogonal.
Sol.

(b) Find the area enclosed by the parabola and the common chord of the ellipse and parabola.
Sol.

(c) If tangent and normal at the point P on the ellipse intersect the x-axis at T and G respectively then find the area of the triangle PTG.
Sol.

10. A normal inclined at 45º to the axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is drawn. It meets the x-axis & the y-axis in P & Q respectively. If C is the centre of the ellipse, show that the area of triangle CPQ is $\frac{(a^2-b^2)^2}{2(a^2+b^2)}$ sq. units.

Sol.

11. Tangents are drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from the point $\left(\frac{a^2}{\sqrt{a^2-b^2}}, \sqrt{a^2+b^2}\right)$. Prove that they intercept on the ordinate through the nearer focus a distance equal to the major axis.
Sol.

12. A straight line AB touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ & the circle $x^2 + y^2 = r^2$; where $a > r > b$. A focal chord of the ellipse, parallel to AB intersects the circle in P & Q, find the length of the perpendicular drawn from the centre of the ellipse to PQ. Hence show that PQ = 2b.

Sol.

13. A ray emanating from the point (– 4, 0) is incident on the ellipse $9x^2 + 25y^2 = 225$ at the point P with abscissa 3. Find the equation of the reflected ray after first reflection.

Sol.

14. If p is the length of the perpendicular from the focus 'S' of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ on any tangent at 'P', then show that $\frac{b^2}{p^2} = \frac{2a}{\ell(SP)} - 1$.

Sol.

15. Variable pairs of chords at right angles and drawn through any point P (with eccentric angle $\pi/4$) on the ellipse $\frac{x^2}{4} + y^2 = 1$, to meet the ellipse at two points say A and B. If the line joining A and B passes through a fixed point Q(a, b) such that $a^2 + b^2$ has the value equal to $\frac{m}{n}$, where m, n are relatively prime positive integers, find (m + n).

Sol.